ESTIMATING TAX BUOYANCY, ELASTICITY AND STABILITY

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ABSTRACT

This methodological note explains how to calculate tax buoyancy and tax elasticity, illustrating the techniques with examples drawn from Madagascar. It develops a measure of tax stability, and shows how to determine when an increase in a given tax leads to greater, or less, overall revenue stability.

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As GDP rises, do tax revenues rise at the same pace? To answer this question it is useful to measure the buoyancy and elasticity of a tax.

Tax buoyancy

Tax (or revenue) buoyancy is defined as

$$TB = \%DRevenue \div \%DBase$$

using numbers for the revenue and base actually observed. Typically the base is taken to be GDP, although other bases are possible (e.g. consumption as the base for sales taxes, imports as the base for tariffs, etc.). The revenue could refer to total tax revenue, or to revenue from any given tax.

The increases are measured in *real* terms - i.e. after adjusting for inflation. If the increases were measured in nominal values then the estimate of TB would be biased towards 1, as the following example illustrates.

> Example: *Suppose that between 1995 and 1996 nominal GDP rises by 20% and the revenue collected by excise taxes on beer rise by 21%. Inflation is estimated at 15%. Calculate the tax buoyancy.*
>
> The first step is to deflate the values. Real GDP rises by 4.35% (= 1.2/1.15-1) and real excise tax revenue rises by 5.22% (=1.21/1.15-1). Thus the TB is 5.22%/4.35% = 1.2. This may be interpreted as indicating that when real GDP rises 1%, excise revenue rises by 1.2%, or 20% more quickly.
>
> Note that if we had not deflated the growth rates, the measure of TB would have been 1.05 (= 21%/20%), which is closer to 1. This measure understates the responsiveness of revenue to a change in real GDP.

As a practical matter, measures of tax buoyancy tend to vary a lot from year to year, which is not very helpful. It is thus more useful to measure buoyancy over a longer period - perhaps five or ten years at a time. There are a number of different ways to do this. Here are some of the more commonly used techniques:

a) *Calculate buoyancy for each year, and then take the average.* This has the disadvantage that it can be heavily influenced by unusually high or low (or negative) measures of tax buoyancy for some of the years, and so is the least satisfactory approach.

b) *Calculate the growth of tax revenue, and of the base (e.g. GDP), between the end years* and use these to calculate buoyancy. The problem here is that the result is sensitive to the end

years cho`sen, but it does have the advantage that one only needs to have data on revenue and GDP for two years (appropriately spaced).

c) *Calculate the growth of tax revenue, and of the base (e.g. GDP), between the average end years* (e.g. the average of the first three years of the series, compared with the last three years of the series). This is less sensitive to the choice of years than the procedure in b, but requires more data.

d) *Regress the log of tax revenue on the year, to get the average growth rate of tax revenue. Do the same for the base (e.g. GDP). The growth rates are the coefficients of the independent variable (the year). Use these growth rates to calculate buoyancy.* This procedure generally yields sensible results, but is least successful in cases where the coefficients in the regressions are not statistically significant or where the growth rate of the base is very small.

e) *Regress the log of tax revenue on the log of the base (e.g. GDP).* The coefficient on the log of the base is a measure of the tax buoyancy. This is an elegant approach, although the results are somewhat sensitive to unusual years (outliers) and to the time interval used in the regression. It also needs data for every year (as do methods a and d).

Application.   *Calculate the buoyancy of total government revenue in Madagascar using data from 1984 to 1995.*

The Appendix Table shows GDP and government revenue (tax revenue, non-tax revenue, and grants) for Madagascar for 1984-1995. Both are in nominal terms. The GDP is converted into real terms using the GDP deflator, while the government revenue series is converted using the consumer price index. We get the following results:

Table 1

Method	Summary of method	Tax buoyancy
a	Average of annual buoyancy measures	27.6
b	Use growth rates between end points	0.43
c	Use growth rates between average end points	1.10
d	Use growth rates based on regressions	0.61
e	Regress Ln(revenue) on Ln(GDP)	1.53

These results differ substantially from one another, but only method a is clearly wrong. Indeed the results are not entirely satisfactory, because it is not even clear whether the tax buoyancy is greater than, or less than, one.

Method d yields a plausible result. It is based on the following two regressions:
```
Ln(GDP)        =      -14.1 + 0.0115 Year
                      p=.009  p=.000        R²=0.72
```
and
```
Ln(gov revenue)=      -7.5 +  0.0070 Year
                      p=.741  p=.543        R²=0.04.
```

This gives a GDP growth rate of 1.15% p.a. (=0.0115) and a growth rate of tax revenue of 0.7% p.a. Thus the revenue buoyancy is 0.61 (=0.0070/0.0115). The main problem here is that the coefficients in the tax revenue equation are not statistically significant, and for all we know they could be zero.

Method e results in the following regression:

```
Ln(gov revenue)=      -6.40 + 1.527 Ln(GDP)
                      p=.290  p=.050          R²=0.33.
```

From this we immediately get the revenue buoyancy of 1.53; this is statistically significantly different from zero (at the 5% level). A scatterplot of the log of real government revenue against the log of real GDP is shown in the top right hand panel of the accompanying figure.

Note that this exercise looks at all government revenue, including grants. It needs to be done for tax revenue only, and for the major categories of tax.
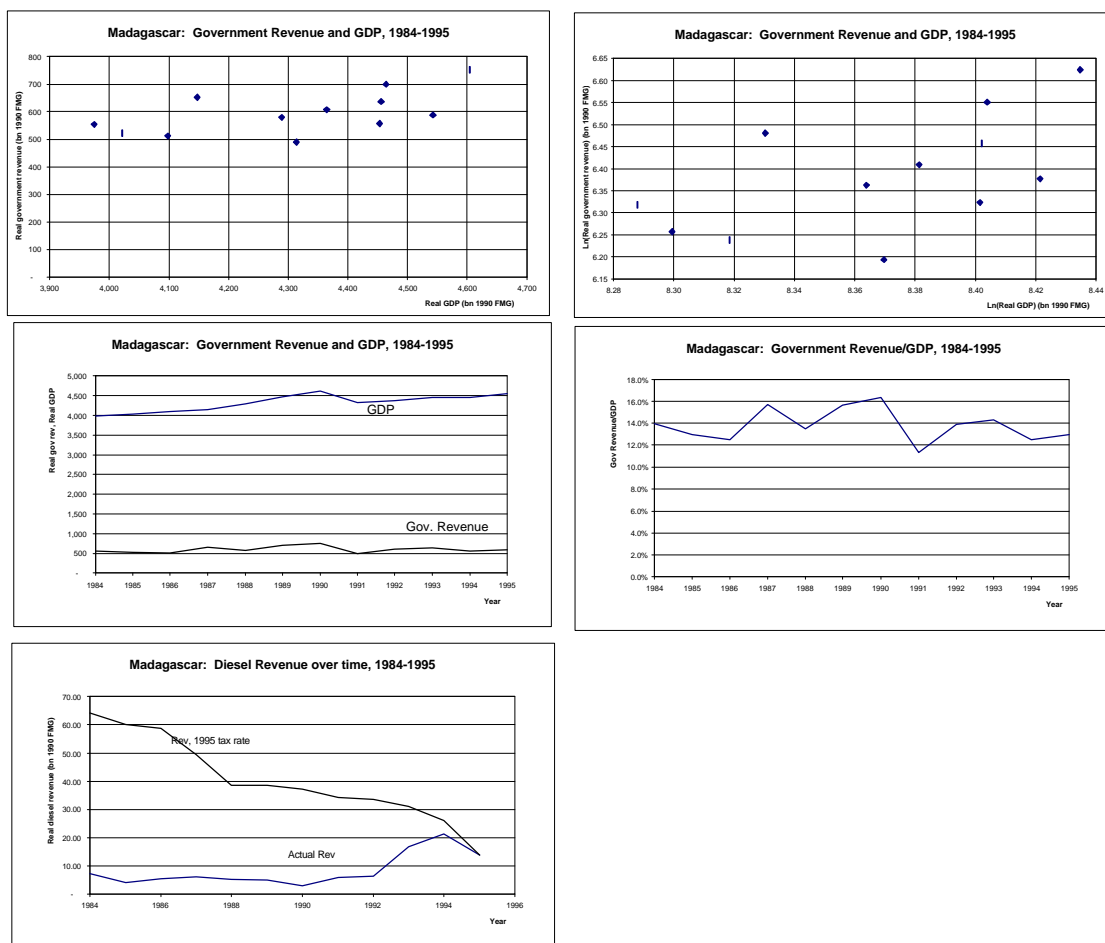
Tax Elasticity

Tax elasticity is defined as

$$TE = \%\mathbf{D}Revenue \div \%\mathbf{D}Base.$$

This looks just like tax buoyancy, but there is a crucial difference, which is that *revenue is calculated as it would have been if there had not been any change in the tax laws, including the tax rates or bases.* Thus the tax elasticity is a hypothetical construct. It tries to reconstruct what would have happened if there had been no changes in the tax rules - i.e. what tax revenue would have been if last year's laws continued to apply this year.

The main use of tax elasticities is to identify which taxes are naturally elastic - i.e. which taxes will yield more revenue as GDP rises, even if the rates are not changed from year to year. Elastic taxes are generally considered to be desirable, because they reduce the need to tinker with the tax system every year. Tax elasticities are unit-free, and so may be compared across countries without any further modification. In the case of Madagascar and Tanzania, it would be helpful to try to estimate tax elasticities for petroleum products, beer, and cigarettes.

Tax elasticities are difficult to construct, because they require one to calculate a counterfactual. They are not usually calculated for total tax revenue, but they can be constructed for individual taxes with some degree of success. The procedure is first to generate a revenue series, assuming no change in the tax rates from year to year; then one may apply methods a-e to compute the elasticity.

Madagascar: Government Revenue and GDP, 1984-1995

Real government revenue (bn 1990 FMG)

Real GDP (bn 1990 FMG)

Madagascar: Government Revenue and GDP, 1984-1995

Ln(Real government revenue) (bn 1990 FMG)

Ln(Real GDP) (bn 1990 FMG)

Madagascar: Government Revenue and GDP, 1984-1995

Real gov rev, Real GDP

GDP

Gov. Revenue

Year

Madagascar: Government Revenue/GDP, 1984-1995

Gov Revenue/GDP

Year

Madagascar: Diesel Revenue over time, 1984-1995

Real diesel revenue (bn 1990 FMG)

Rev, 1995 tax rate

Actual Rev

Year

Example.	*Suppose that in 1997 the tax on beer was 200FMG/liter and 8m liters were sold, yielding a revenue of 1,600mFMG. In 1998 the tax is raised to 240FMG/liter and 8.1m liters are sold, for a revenue of 1,944mFMG. Inflation is running at 15% annually and real GDP is rising by 2.5%. Calculate the buoyancy and elasticity of the beer tax.*

Tax buoyancy.
Revenue in 1997: 1,600. Revenue in 1997, adjusted for inflation: 1944/1.15 = 1690. So increase in real revenue is 5.625%. Since real GDP rose by 2.5% during the same period, this gives a tax buoyancy of 2.25 (=5.625%/2.5%).

Tax elasticity.
The increase in revenue is due both to higher sales of beer, and to the change in the tax law. What would have happened to revenue if the tax of 1997 had not been changed? Presumably the revenue in 1998 would have been 1,620mFMG (=200x8.1m); deflated, this represents 1,409mFMG (=1620/1.15) in 1997 prices, or a reduction of 11.9%. The tax elasticity would therefore be −4.76 (= −11.9/2.5). This indicates that if the tax rate had not been changed, then real revenue would have fallen between 1997 and 1998, despite the increase in GDP. Negative tax elasticities are not unusual where taxes are specific (rather than ad valorem) and where there is a significant level of inflation; in such

cases the specific tax rates need to be adjusted regularly to account for inflation. The measures of tax elasticity are not particularly useful in these situations.

As with measures of tax buoyancy, it is not wise to pay too much attention to the tax elasticity between one year and the next. One needs to consider the measure over a longer period of time - at least five years.

Tax Stability

The revenue from different taxes varies from year to year. Taxes whose revenue is relatively stable, or whose revenue is negatively correlated with the revenue from other taxes, are likely to be particularly helpful in giving stability to the overall stream of revenue. Revenue stability is desirable, at least from the government's perspective, in that it makes it easier to put together plausible spending and borrowing plans for the year ahead.

A simple measure of the stability of tax revenue is the coefficient of variation (CV), which is defined as the standard deviation of tax revenue (as a fraction of GDP usually) divided by its mean; i.e.

Coefficient of Variation = Standard Deviation ¸ Mean.

The CV may be calculated for tax revenue as a whole, or for individual sources of revenue. The measure is most useful when compared across countries.

> Application. *Calculate the coefficient of variation for total government revenue in Madagascar over the period 1984-1995.*
>
> During this period, government revenue in Madagascar (including tax revenue, non-tax revenue and grants) averaged 13.5% of GDP. The standard deviation of this proportion was 1.8%, giving a coefficient of variation of 13.1% (= 1.8%/13.5%).

It is also helpful to know whether revenue from a given tax leads to more, or less, stability in overall tax revenue. If the tax revenue from, say, cigarettes is negatively correlated with revenue from all other sources, then emphasizing cigarette taxes will tend to stabilize total government revenue. One way to measure the impact of a tax on revenue stability is to calculate the coefficient of variation of revenue with, and without, the tax in question. This revenue stabilizing coefficient (RSC) may be defined as

$$RSC1 = CV(total\ revenue\ without\ the\ tax) - CV(total\ revenue\ with\ the\ tax).$$

This coefficient may also be used if the question under consideration is whether an increase in a given tax would lead to more, or less, overall revenue stability. Then we would have

$$RSC2 = CV(total\ revenue\ without\ tax\ increase) - CV(total\ revenue\ with\ tax\ increase).$$

Let R be total revenue, R0 be revenue (from other tax sources) without the tax under consideration, and RT be the revenue from the tax under consideration. We have

$$R = R0 + RT$$

so

$$Var(R) = Var(R0) + Var(RT) + 2Cov(R0,RT)$$

and therefore

$$CV(R)^2 = a^2CV(R0)^2 + (1-a)^2CV(RT)^2 + 2a(1-a)CV(R0)CV(RT)Corr(R0,RT)$$

where $\alpha = (R0/R)$ and Corr(R0,RT) is the correlation coefficient for R0 and RT. This equation is particularly useful when considering the effect of an increase in the tax rate; see below for an example. A tax is more likely to help stabilize total revenue if its own revenue is stable (i.e. CV(RT) is low) and if it is negatively correlated with other sources of revenue (i.e. Corr(R0,RT)<0). We then have the revenue stabilizing coefficient

$$RSC1 = CV(R0)-CV(R).$$

Example.   *Suppose total revenue without sales tax is 900 (SD = 180), and sales tax yields a further 100 (SD = 22) in revenue. The correlation coefficient between sales tax revenue and other revenue is -0.2. Does the sales tax revenue help stabilize total revenue?*

Calculate the RSC1 = CV(all tax) - CV(tax without the sales tax). We have
$$CV(R)^2 = (900/1000)^20.2^2 + (100/1000)^20.22^2$$
$$+ 2(900/1000)(100/1000)(0.2)(0.22)(-0.2) = 0.0313$$
so CV(R) = 0.1769. Thus
$$RSC1 = 0.1769 - 0.2 = -0.023.$$
Although the sales tax has a higher CV (at 0.22) than total revenue (at 0.2), the inclusion of the sales tax leads to greater revenue stability, lowering it from 0.2 to 0.1769, or by almost 12%.

Example.   *Suppose total revenue without sales tax is 900 (SD = 180), and sales tax yields a further 100 (SD = 10) in revenue. The correlation coefficient between sales tax revenue and other revenue is 0.1. Does the sales tax revenue help stabilize total revenue?*

In this case

$$CV(R)^2 = (900/1000)^2(0.2)^2 + (100/1000)^2(0.1)^2$$
$$+ 2(900/1000)(100/1000)(0.2)(0.1)(0.1)$$

so CV(R) = 0.1813 and RSC1 = 0.1813 - 0.2 = 0.0187. Although sales tax revenue is positively correlated with total revenue, this tax contributes to overall revenue stability because its own revenue stream is relatively stable (with a CV of 0.1).

Application.

Comment on the buoyancy, elasticity and stability of the tax on diesel fuel in Madagascar. for 1984-1995.

Buoyancy.
Using method d we get:
```
Ln(GDP)        =      -14.1 + 0.0115 Year
                      p=.009  p=.000        R²=0.72
```
and
```
Ln(diesel revenue)=   -205 + 0.104 Year
                      p=.029  p=.028        R²=0.40.
```
This gives a buoyancy of 9.0, which seems rather high. Method e results in the following regression:
```
Ln(diesel revenue)=   -24.81 +3.20 Ln(GDP)
                      p=.44   p=.41         R²=0.07.
```
From this we get the revenue buoyancy of 3.2, but it is not statistically significant.

Elasticity
We first recompute diesel fuel tax revenue for every year, assuming that the 1995 rate (of 345FMG/liter) applied in every year. This artificial series, as well as actual revenue, are graphed in the bottom left panel of the Figure. We then regressed the artificial ("constant tax") series on time, to get an annual growth rate of −10.9% p.a.; in other words, if there had been no changes in the tax rate, revenue from tax on diesel fuel would have fallen by 10.9% annually. Since GDP grew by 1.15% p.a. during the period, we have a diesel tax elasticity of −9.5.

Stability
We have the following information, based on the revenue data:

	Revenue		
	Total	Non-Diesel	Diesel
Mean	595.94	587.63	8.30
SD	75.22	76.12	5.51
CV	0.1262	0.1295	0.6631
Correlation			-0.1988

Real revenue from the TUPP on diesel fuel has a high CV, but is negatively correlated with real revenue from other sources. The net effect is that this tax contributes to the overall stability of revenue. We have
$$CV(R)^2 = (587.94/595.94)^2(0.1295)^2 + (8.30/595.94)^2(0.6631)^2$$
$$+ 2(587.94/595.94)(8.30/595.94)(0.1295)(0.6631)(-.1988)$$
$$= .1262^2.$$
In other words, the CV of the tax system without the diesel tax is 0.1295, but with the tax is 0.1262.

What if the diesel tax were doubled, while other taxes were reduced, so that revenue remains as before. Then we would have

$$CV(R)^2 = (579.64/595.94)^2(0.1295)^2 + (16.6/595.94)^2(0.6631)^2$$
$$+ 2(579.64/595.94)(16.6/595.94)(0.1295)(0.6631)(-.1988)$$
$$= .1236^2.$$

That is, the CV for all revenue would be reduced to .1236 in this case.

Appendix Table. Government Revenue, GDP and Prices for Madagascar, 1984-1995.

	GDP	GDP deflator	Real GDP	Ln real GDP	Govt Revenue	CPI	Real G Revenue	Ln real G Revenue	Real Rev / GDP	Diesel Tax/liter	Diesel Revenue	Real Diesel Revenue	Ln(real diesel Revenue)	Revenue at 1995 rate	Real rev at 1995 rate	Ln(real rev)
1984	1,695	100	3,975	8.29	243	43.9	554	6.32	13.9%	21.87	3.20	7.30	1.99	28.15	64.09	4.16
1985	1,893	110	4,021	8.30	252	48.3	522	6.26	13.0%	12.89	1.94	4.02	1.39	28.93	59.94	4.09
1986	2,204	126	4,098	8.32	280	54.8	511	6.24	12.5%	17.92	3.01	5.49	1.70	32.24	58.82	4.07
1987	2,743	155	4,148	8.33	421	64.5	652	6.48	15.7%	23.45	3.89	6.03	1.80	31.87	49.40	3.90
1988	3,437	188	4,289	8.36	474	81.7	580	6.36	13.5%	25.73	4.21	5.16	1.64	31.43	38.48	3.65
1989	4,005	210	4,464	8.40	625	89.3	700	6.55	15.7%	24.70	4.42	4.95	1.60	34.37	38.49	3.65
1990	4,604	235	4,604	8.43	753	100.0	753	6.62	16.4%	15.00	2.90	2.90	1.06	37.11	37.11	3.61
1991	4,914	267	4,314	8.37	533	108.9	490	6.19	11.3%	32.50	6.32	5.81	1.76	37.38	34.32	3.54
1992	5,593	301	4,365	8.38	753	123.9	607	6.41	13.9%	35.42	7.69	6.20	1.83	41.70	33.64	3.52
1993	6,451	340	4,456	8.40	864	135.7	637	6.46	14.3%	102.92	22.58	16.65	2.81	42.15	31.07	3.44
1994	9,131	481	4,454	8.40	1,036	185.9	557	6.32	12.5%	156.67	39.73	21.38	3.06	48.71	26.21	3.27
1995	13661.6	705.2	4,543	8.42	1,623	276	588	6.38	12.9%	192.08	37.99	13.76	2.62	37.99	13.76	2.62

Source: Pépé Andrianomanana and Jean Razafindravonona. 1997. Data file on excise taxes in Madagascar. EAGER/PSG Excise Project.